Find the first & second derivatives:

① $x^2 \sin(x)$

② $(3x-4)e^{-x^2/10}$

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① $[x^2 \sin(x)]' = 2x \sin(x) + x^2 \cos(x).$

$[x^2 \sin(x)]'' = 2\sin(x) + 2x\cos(x) + 2x\cos(x) - x^2 \sin(x)$

$= (2-x^2)\sin(x) + 4x\cos(x)$

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② $\left[(3x-4)e^{-x^2/10}\right]' = 3e^{-x^2/10} + (3x-4)\left[e^{-x^2/10}\right]'$

$= 3e^{-x^2/10} + (3x-4)\, e^{-x^2/10} \cdot \left(\frac{-1}{10}\cdot 2x\right)$

$= 3e^{-x^2/10} + \left(3xe^{-x^2/10} - 4e^{-x^2/10}\right)\left(\frac{-1}{5}x\right)$

$= 3e^{-x^2/10} + \frac{-3}{5}x^2 e^{-x^2/10} + \frac{4}{5}x e^{-x^2/10}$

$= \boxed{\left(\frac{-3}{5}x^2 + \frac{4}{5}x + 3\right)e^{-x^2/10}}$

$\left[(3x-4)\, e^{-x^2/10}\right]'' = \left[\left(\frac{-3}{5}x^2 + \frac{4}{5}x + 3\right)e^{-x^2/10}\right]'$

$= \left(\frac{-6}{5}x + \frac{4}{5}\right)\cdot e^{-x^2/10} + \left(\frac{-3}{5}x^2 + \frac{4}{5}x + 3\right)\cdot e^{-x^2/10} \cdot \left(\frac{-1}{5}x\right)$

$= e^{-x^2/10} \cdot \left(\frac{-6}{5}x + \frac{4}{5} + \frac{3}{25}x^3 - \frac{4}{25}x^2 - \frac{3}{5}x\right)$

$$= \boxed{e^{-x^2/10} \cdot \left(\frac{3}{25}x^3 - \frac{4}{25}x^2 - \frac{9}{5}x + \frac{4}{5}\right)}$$

Finding absolute (global) maxima & minima.

"extrema" - means either a max or min.

Example: Find the absolute maximum & minimum of the function $y = x\left(x-\frac{\pi}{2}\right)\tan(x)$ on the interval

a) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

b) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right]$

c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

a) Let's find interior critical points:

$y' = 0$

$$y' = \left(x-\frac{\pi}{2}\right)\tan(x) + x\tan(x) + x\left(x-\frac{\pi}{2}\right)\cdot\sec^2(x)$$

$$= \left(2x-\frac{\pi}{2}\right)\tan(x) + x\left(x-\frac{\pi}{2}\right)\sec^2(x)$$

$$= \frac{\left(2x-\frac{\pi}{2}\right)\sin(x)}{\cos(x)} + \frac{x\left(x-\frac{\pi}{2}\right)}{\cos^2(x)}$$

$$= \frac{\left(2x-\frac{\pi}{2}\right)\sin(x)\cos(x) + x\left(x-\frac{\pi}{2}\right)}{\cos^2(x)} = 0$$

$$\left(2x-\frac{\pi}{2}\right)\sin(x)\cos(x) + x\left(x-\frac{\pi}{2}\right) = 0$$

Guesses for solutions: $x=0$ ✓

$x=\frac{\pi}{2}$. ✓

let's graph the numerator.

← only solutions in intervals.

ⓐ $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$: $x=0$ $y = x\left(x-\frac{\pi}{2}\right)\tan(x)$

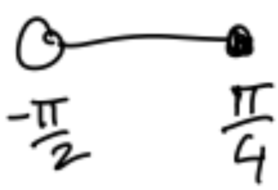
$= 0\cdot\left(-\frac{\pi}{2}\right)\cdot 0 =$

cr pts:

	x	$f(x)$
abs max →	$x=0$	$y=0$
end pts		
abs min →	$x=-\frac{\pi}{4}$	$y=-\frac{\pi}{4}\left(-\frac{3\pi}{4}\right)(-1) = -\frac{3\pi^2}{16}$.
	$x=\frac{\pi}{4}$	$y=\frac{\pi}{4}\left(-\frac{\pi}{4}\right)1 = \frac{-\pi^2}{16}$.

ⓑ $\left[-\frac{\pi}{2}, \frac{\pi}{4}\right]$

	x	$f(x)$
cr pt:	$x=0$	$y=0$
end pt:	$x=\frac{\pi}{4}$	$y=\frac{-\pi^2}{16}$



What about $\lim_{x \to -\frac{\pi}{2}^+} f(x) = ?$

$$\lim_{x \to -\frac{\pi}{2}^+} \frac{x\,(x-\frac{\pi}{2})\sin(x)}{\cos(x)} = -\infty$$

$-\frac{\pi}{2}$, $-\pi$, $\to -1$, $\to 0^+$

Numerator $\to \frac{-\pi^2}{2}$

denom $\to 0^+$

$\boxed{\text{No abs min!}}$

$\boxed{\therefore\ x=0 \text{ abs max}}$

ⓒ $\left(-\frac{\pi}{2}, +\frac{\pi}{2}\right)$.

cr pts:

x	$f(x)$
$x=0$	$y=0$
no end pts.	

$$\lim_{x \to -\frac{\pi}{2}^+} f(x) = -\infty$$

$$\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} \frac{x\left(x-\frac{\pi}{2}\right)\sin(x)}{\cos(x)}$$

$\frac{0}{0}$ form

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{x\,(x-\frac{\pi}{2})\sin(x)}{-\sin(x-\frac{\pi}{2})}$$

θ, θ, $\theta \to 0$, $\to 1$

Note

$\sin(x-\frac{\pi}{2})$

$= \sin(x)\underbrace{\cos(-\tfrac{\pi}{2})}_{0} + \cos(x)\underbrace{\sin(-\tfrac{\pi}{2})}_{-1}$

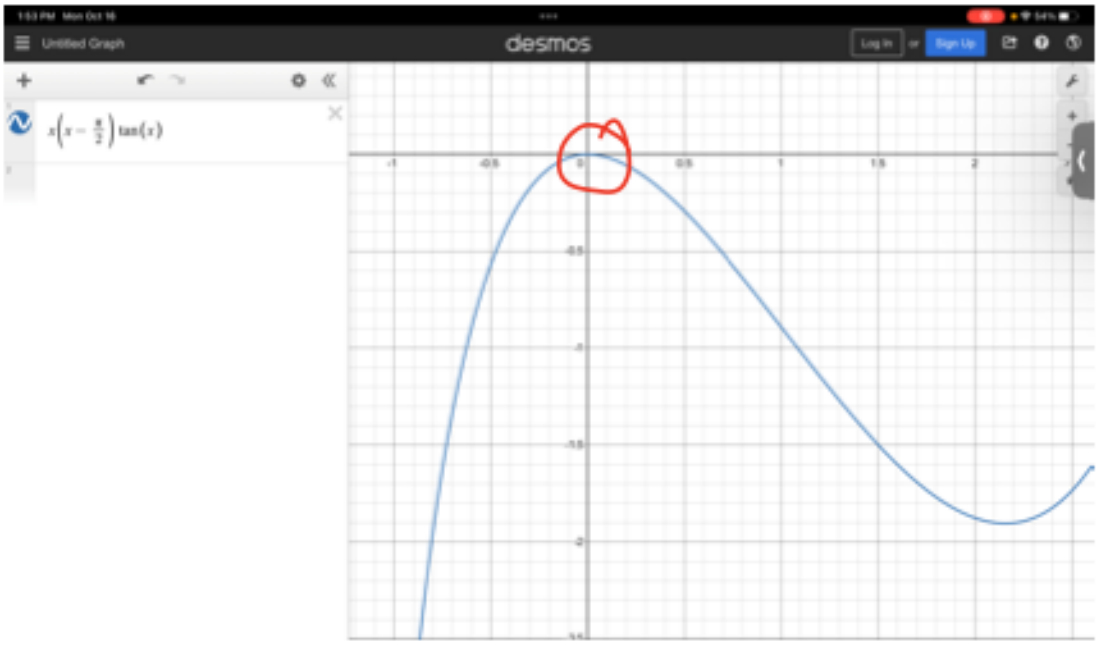
$= -\cos(x)$.

$$= \left(\frac{\pi}{2}\right)\cdot 1 \cdot \frac{\sin(\pi/2)}{-1} = \boxed{-\frac{\pi}{2}}$$

$$\lim_{x \to \frac{\pi}{2}^-} = -\frac{\pi}{2}$$

$\therefore$ $x=0$ is abs max

no global min



Yep.